

3-2 Study Guide and Intervention

Solving Systems of Equations Algebraically

Substitution To solve a system of linear equations by **substitution**, first solve for one variable in terms of the other in one of the equations. Then substitute this expression into the other equation and simplify.

Example

Use substitution to solve the system of equations.

$$\begin{aligned} 2x - y &= 9 \\ x + 3y &= -6 \end{aligned}$$

Solve the first equation for y in terms of x .

$$\begin{aligned} 2x - y &= 9 && \text{First equation} \\ -y &= -2x + 9 && \text{Subtract } 2x \text{ from both sides.} \\ y &= 2x - 9 && \text{Multiply both sides by } -1. \end{aligned}$$

Substitute the expression $2x - 9$ for y into the second equation and solve for x .

$$\begin{aligned} x + 3y &= -6 && \text{Second equation} \\ x + 3(2x - 9) &= -6 && \text{Substitute } 2x - 9 \text{ for } y. \\ x + 6x - 27 &= -6 && \text{Distributive Property} \\ 7x - 27 &= -6 && \text{Simplify.} \\ 7x &= 21 && \text{Add } 27 \text{ to each side.} \\ x &= 3 && \text{Divide each side by } 7. \end{aligned}$$

Now, substitute the value 3 for x in either original equation and solve for y .

$$\begin{aligned} 2x - y &= 9 && \text{First equation} \\ 2(3) - y &= 9 && \text{Replace } x \text{ with } 3. \\ 6 - y &= 9 && \text{Simplify.} \\ -y &= 3 && \text{Subtract } 6 \text{ from each side.} \\ y &= -3 && \text{Multiply each side by } -1. \end{aligned}$$

The solution of the system is $(3, -3)$.

Exercises

Solve each system of equations by using substitution.

1. $\begin{cases} 3x + y = 7 \\ 4x + 2y = 16 \end{cases}$

2. $\begin{cases} 2x + y = 5 \\ 3x - 3y = 3 \end{cases}$

3. $\begin{cases} 2x + 3y = -3 \\ x + 2y = 2 \end{cases}$

4. $\begin{cases} 2x - y = 7 \\ 6x - 3y = 14 \end{cases}$

5. $\begin{cases} 4x - 3y = 4 \\ 2x + y = -8 \end{cases}$

6. $\begin{cases} 5x + y = 6 \\ 3 - x = 0 \end{cases}$

7. $\begin{cases} x + 8y = -2 \\ x - 3y = 20 \end{cases}$

8. $\begin{cases} 2x - y = -4 \\ 4x + y = 1 \end{cases}$

9. $\begin{cases} x - y = -2 \\ 2x - 3y = 2 \end{cases}$

10. $\begin{cases} x - 4y = 4 \\ 2x + 12y = 13 \end{cases}$

11. $\begin{cases} x + 3y = 2 \\ 4x + 12y = 8 \end{cases}$

12. $\begin{cases} 2x + 2y = 4 \\ x - 2y = 0 \end{cases}$

3-2 Study Guide and Intervention *(continued)***Solving Systems of Equations Algebraically**

Elimination To solve a system of linear equations by **elimination**, add or subtract the equations to eliminate one of the variables. You may first need to multiply one or both of the equations by a constant so that one of the variables has the opposite coefficient in one equation as it has in the other.

Example 1 Use the elimination method to solve the system of equations.

$$2x - 4y = -26$$

$$3x - y = -24$$

Multiply the second equation by -4 . Then add the equations to eliminate the y variable.

$$\begin{array}{r} 2x - 4y = -26 \\ 3x - y = -24 \quad \text{Multiply by } -4 \rightarrow \end{array} \begin{array}{r} 2x - 4y = -26 \\ -12x + 4y = 96 \\ \hline -10x \qquad = 70 \\ x \qquad = -7 \end{array}$$

Replace x with -7 and solve for y .

$$2x - 4y = -26$$

$$2(-7) - 4y = -26$$

$$-14 - 4y = -26$$

$$-4y = -12$$

$$y = 3$$

The solution is $(-7, 3)$.

Example 2 Use the elimination method to solve the system of equations.

$$3x - 2y = 4$$

$$5x + 3y = -25$$

Multiply the first equation by 3 and the second equation by 2 . Then add the equations to eliminate the y variable.

$$\begin{array}{r} 3x - 2y = 4 \\ 5x + 3y = -25 \end{array} \begin{array}{r} \text{Multiply by } 3 \rightarrow \\ \text{Multiply by } 2 \rightarrow \end{array} \begin{array}{r} 9x - 6y = 12 \\ 10x + 6y = -50 \\ \hline 19x \qquad = -38 \\ x \qquad = -2 \end{array}$$

Replace x with -2 and solve for y .

$$3x - 2y = 4$$

$$3(-2) - 2y = 4$$

$$-6 - 2y = 4$$

$$-2y = 10$$

$$y = -5$$

The solution is $(-2, -5)$.

Exercises

Solve each system of equations by using elimination.

1. $2x - y = 7$
 $3x + y = 8$

2. $x - 2y = 4$
 $-x + 6y = 12$

3. $3x + 4y = -10$
 $x - 4y = 2$

4. $3x - y = 12$
 $5x + 2y = 20$

5. $4x - y = 6$
 $2x - \frac{y}{2} = 4$

6. $5x + 2y = 12$
 $-6x - 2y = -14$

7. $2x + y = 8$
 $3x + \frac{3}{2}y = 12$

8. $7x + 2y = -1$
 $4x - 3y = -13$

9. $3x + 8y = -6$
 $x - y = 9$

10. $5x + 4y = 12$
 $7x - 6y = 40$

11. $-4x + y = -12$
 $4x + 2y = 6$

12. $5m + 2n = -8$
 $4m + 3n = 2$