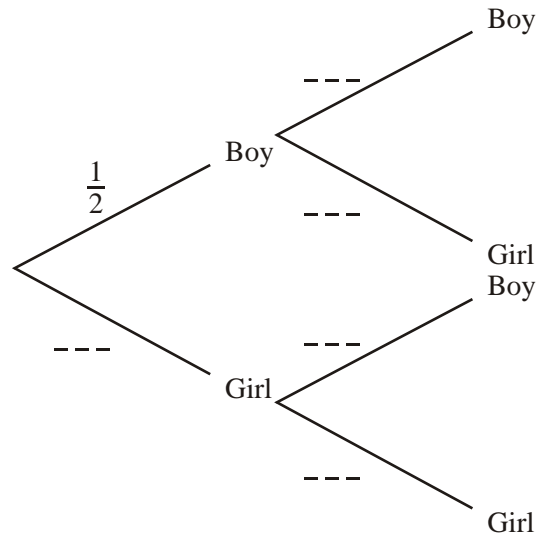


1. Let F be the set of all families that have exactly 2 children.

(a) Assuming $P(\text{boy}) = P(\text{girl})$, copy and complete the following tree diagram, for families with 2 children.



(2)

(b) What is the probability that a family chosen at random from F has exactly

- (i) 2 boys?
- (ii) 2 boys, if it is known that the first child is a boy?
- (iii) 2 boys, if it is known that there is a boy in the family?

(3)

(Total 5 marks)

2. Let p and q be the statements

p : you watch the music TV channel

q : you like music

(a) Consider the following logic statement.

If you watch the music TV channel then you like music.

- (i) Write down in words the inverse of the statement.
- (ii) Write down in words the converse of the statement.

(4)

(b) Construct truth tables for the following statements:

- (i) $p \Rightarrow q$.
- (ii) $\neg p \Rightarrow \neg q$.

(iii) $p \vee \neg q$.

(iv) $\neg p \wedge q$.

(4)

(c) Which of the statements in part (b) are logically equivalent?

(1)

(Total 9 marks)

3. Let p and q be the statements

p : you watch the music TV channel

q : you like music

(a) Consider the following logic statement.

If you watch the music TV channel then you like music.

(i) Write down in words the inverse of the statement.

(ii) Write down in words the converse of the statement.

(4)

(b) Construct truth tables for the following statements:

(i) $p \Rightarrow q$.

(ii) $\neg p \Rightarrow \neg q$.

(iii) $p \vee \neg q$.

(iv) $\neg p \wedge q$.

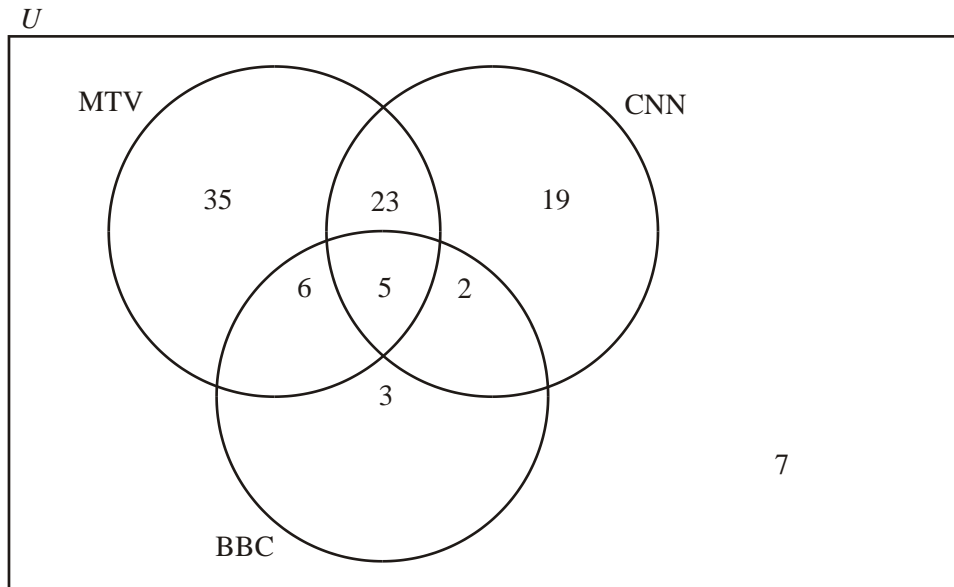
(4)

(c) Which of the statements in part (b) are logically equivalent?

(1)

(Total 9 marks)

4. 100 students were asked which television channel (MTV, CNN or BBC) they had watched the previous evening. The results are shown in the Venn diagram below.



From the information in the Venn diagram, write down the number of students who watched

- both MTV and BBC;
- MTV or BBC;
- CNN and BBC but not MTV;
- MTV or CNN but not BBC.

(Total 4 marks)

5. Let the propositions p , q and r be defined as:

p : Matthew arrives home before six o'clock

q : Matthew cooks dinner

r : Jill washes the dishes

- (i) Express the following statement in logical form.

If Matthew arrives home before six o'clock then he will cook dinner.

(1)

- (ii) Write the following logic statement in words.

$$\neg q \Rightarrow \neg r$$

(1)

(b) (i) Copy and complete the truth table below.

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$\neg r$	$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r$	$\neg p$	$[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r] \Rightarrow \neg p$
T	T	T						T
T	T	F						T
T	F	T						T
T	F	F						T
F	T	T						T
F	T	F						T
F	F	T						T
F	F	F						T

(5)

(ii) Explain the significance of the truth table above.

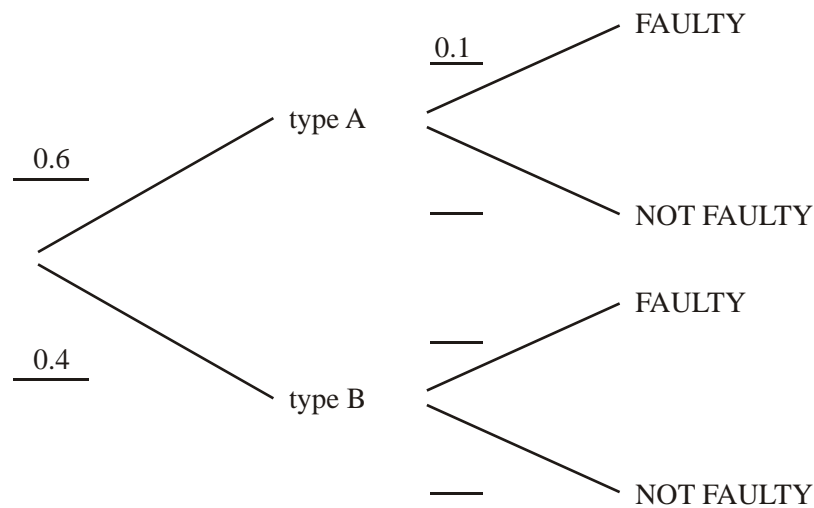
(2)

(Total 9 marks)

6. A teacher has a box containing six type A calculators and four type B calculators.

The probability that a type A calculator is faulty is 0.1 and the probability that a type B calculator is faulty is 0.12.

(a) Complete the tree diagram given below, showing all the probabilities.



- (b) A calculator is selected at random from the box. Find the probability that the calculator is
- (i) a faulty type A;
 - (ii) not faulty.

Working:

Answers:

(b) (i)

(ii)

(Total 4 marks)

7. Two propositions p and q are defined as follows:

p : the number ends in zero q : the number is divisible by 5

- (a) Write in words

(i) $p \Rightarrow q$;

(ii) the converse of $(p \Rightarrow q)$.

- (b) Write in symbolic form
- (i) the inverse of $(p \Rightarrow q)$;
 - (ii) the contrapositive of $(p \Rightarrow q)$.

Working:

Answers:

- (a) (i)
-
- (ii)
-
- (b) (i)
- (ii)

(Total 4 marks)

8. Two propositions p and q are defined as follows.

p : Jones passed this course
q : Smith passed this course

- (a) Write in symbolic form
- (i) *neither Jones nor Smith passed the course;*
 - (ii) *it is not the case that Jones and Smith both passed the course.*

(b) Complete the following truth table for the logic statement $\neg p \vee q$.

p	q	$\neg p$	$\neg p \vee q$
T	T		
T	F		
F	T		
F	F		

Working:

Answers:

(a) (i)

(ii)

(Total 4 marks)

9. The sets A, B and C are subsets of U . They are defined as follows:

$$U = \{\text{positive integers less than } 16\}$$

$$A = \{\text{prime numbers}\}$$

$$B = \{\text{factors of } 36\}$$

$$C = \{\text{multiples of } 4\}$$

(a) List the elements (if any) of the following:

(i) A ;

(ii) B ;

(iii) C ;

(iv) $A \cap B \cap C$.

(4)

(b) (i) Draw a Venn diagram showing the relationship between the sets U, A, B and C .

(ii) Write the elements of sets U, A, B and C in the appropriate places on the Venn diagram.

(4)

(c) From the Venn diagram, list the elements of each of the following

(i) $A \cap (B \cup C)$;

(ii) $(A \cap B)'$;

(iii) $(A \cap B)' \cap C$.

(3)

(d) Find the probability that a number chosen at random from the universal set U will be

(i) a prime number;

(ii) a prime number, but **not** a factor of 36;

(iii) a factor of 36 or a multiple of 4, but **not** a prime number;

(iv) a prime number, given that it is a factor of 36.

(6)

(Total 17 marks)

10. $[(p \Leftrightarrow q) \wedge p] \Rightarrow q$

(a) Complete the truth table below for the compound statement above.

p	q	$p \Leftrightarrow q$	$(p \Leftrightarrow q) \wedge p$	$[(p \Leftrightarrow q) \wedge p] \Rightarrow q$
T	T			
T	F			
F	T			
F	F			

(b) Explain the significance of your result.

Working:

Answers:

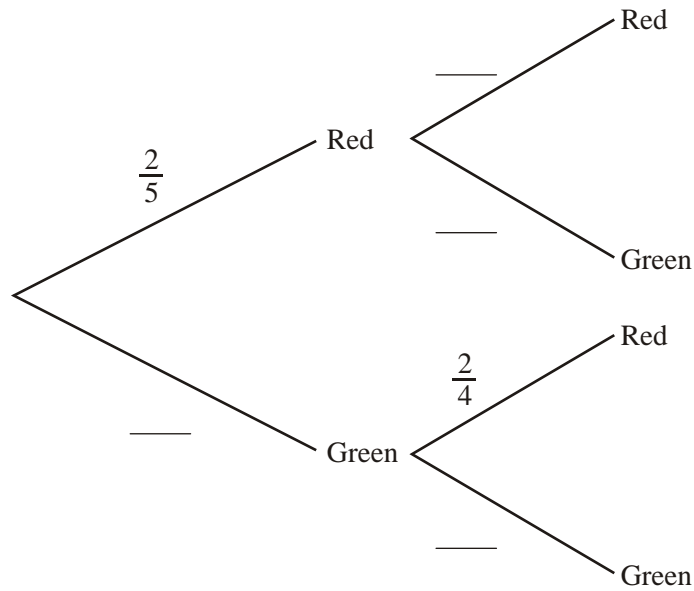
(b)

.....

(Total 4 marks)

11. A bag contains two red sweets and three green sweets. Jacques takes one sweet from the bag, notes its colour, then eats it. He then takes another sweet from the bag.

Complete the tree diagram below to show all probabilities.



Working:

(Total 4 marks)

12. In a survey of 52 students it was found that 30 study Spanish and 15 have computers. Seven of the students who study Spanish also have computers.

(a) **Copy** and complete this table.

	Study Spanish	Do not study Spanish	Total
Have computers			
Do not have computers			
Total			52

- (b) Draw and **label fully** a Venn diagram to illustrate this information. Use U to represent the set of all students surveyed, S the set of students who study Spanish and C the set of students who have computers. (2)
- (c) Describe, in words, the set represented by $C \cup S'$. (2)
- (d) Find $n(C \cup S')$. (1)

A student is selected at random to attend a computer workshop given in Spanish.

- (e) What is the probability that the student
- (i) has a computer and studies Spanish? (2)
- (ii) as a computer but does not study Spanish? (2)
- (iii) as a computer if he/she studies Spanish? (2)
- (Total 14 marks)**

13. If each of the following compound propositions is true, what conclusions can be made?

- (a) $x < 7$ or $x \geq 3$, and $x \not< 7$
- (b) $p = 3$ if and only if $q = 5$, and if $q \neq 5$ then $r \neq 12$.

Working:

Answers:

(a)

(b)

(Total 4 marks)

14. The universal set U is defined as the set of positive integers less than 10. The subsets A and B are defined as:

$$A = \{\text{integers that are multiples of 3}\}$$

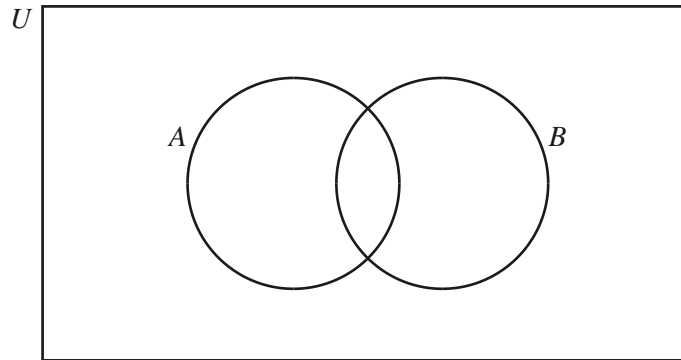
$$B = \{\text{integers that are factors of 30}\}$$

- (a) List the elements of

(i) A ;

(ii) B .

- (b) Place the elements of A and B in the appropriate region in the Venn diagram below.



Working:

Answers:

(a) (i)

(ii)

(Total 4 marks)

15. Members of a certain club are required to register for one of three games, billiards, snooker or darts.

The number of club members of each gender choosing each game in a particular year is shown in the table below.

	Billiards	Snooker	Darts
Male	39	16	8
Female	21	14	17

- (a) Use a χ^2 (Chi-squared) test at the 5% significance level to test whether choice of games is independent of gender. State clearly the null and alternative hypotheses tested, the expected values, and the number of degrees of freedom used.

(13)

The following year the choice of games was widened and the figures for that year are as follows:

	Billiards	Snooker	Darts	Fencing
Male	4	15	8	10
Female	10	21	17	37

- (b) If the χ^2 test were applied to this new set of data,
- (i) why would it be necessary to combine billiards with another game?
 - (ii) which other game would you combine with billiards and why?

(2)

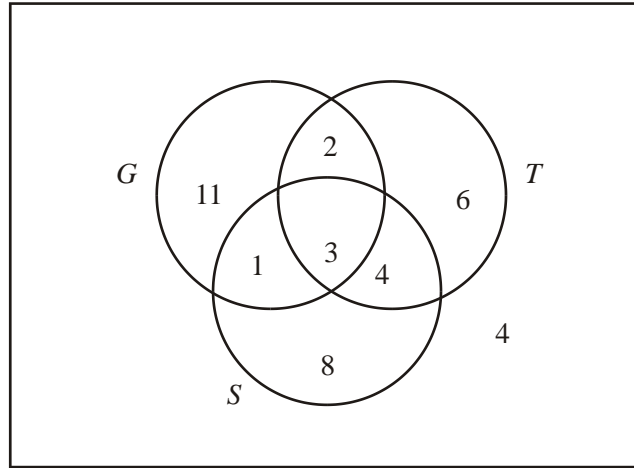
A club member is to be selected at random.

- (c) What is the probability that the club member selected is a
- (i) female who chose billiards or snooker?
 - (ii) male or female who chose darts or fencing?

(2)

(Total 17 marks)

16. The sports offered at a retirement village are Golf (G), Tennis (T) and Swimming (S). The Venn diagram shows the numbers of people involved in each activity.



- (a) How many people
- only play golf?
 - play both tennis and golf?
 - do not play golf?
- (b) Shade the part of the Venn diagram that represents the set $\complement G \cap S$.

Working:

Answers:

- (a) (i)
- (ii)
- (iii)

(Total 4 marks)

17. Two jars contain a number of coloured balls as indicated in the diagrams below.



Jar One



Jar Two

Two experiments are carried out.

First Experiment: A jar is first chosen at random and then a ball is drawn from that jar.

- (a) Draw, **and label fully**, a tree diagram to show **all** possible outcomes of this experiment. (2)
- (b) What is the probability that a white ball is drawn? (3)

Second Experiment: The ball drawn in the first experiment is not replaced. A second ball is then drawn from the same jar.

- (c) What is the probability that both balls are white? (2)
- (Total 7 marks)

18. The propositions p and q are defined as follows:

p : you have understood this topic

q : you will be able to do this question

- (a) Write the following proposition in symbols using p , q and logical connectives only.

'You have understood this topic, or you will not be able to do this question.'

(b) Explain, in words only, what the following symbolic proposition represents:

$$(p \wedge \neg q) \Rightarrow \neg p.$$

Working:

Answers:

(a)

(b)

(Total 4 marks)

19. The propositions p , q and r are defined as follows:

p : *this is a good course*

q : *the course is worth taking*

r : *the grading is lenient*

(a) Write a symbolic statement for each of the following sentences.

(i) *If this is a good course, then it is worth taking.*

(ii) *Either the grading is lenient, or the course is not worth taking.*

(2)

(b) Write the following argument using p , q , r and logic symbols or connectives only.

If this is a good course, then it is worth taking. Either the grading is lenient, or the course is not worth taking. But the grading is not lenient. Therefore, this is not a good course.

(2)

(Total 4 marks)

20. Nene and Deka both play netball. The probability that Nene will score a goal on her first attempt is 0.75. The probability that Deka will score a goal on her first attempt is 0.82.

Calculate the probability that

(a) Nene and Deka will both score a goal on their first attempts;

(b) neither Nene nor Deka will score a goal on their first attempts.

Working:

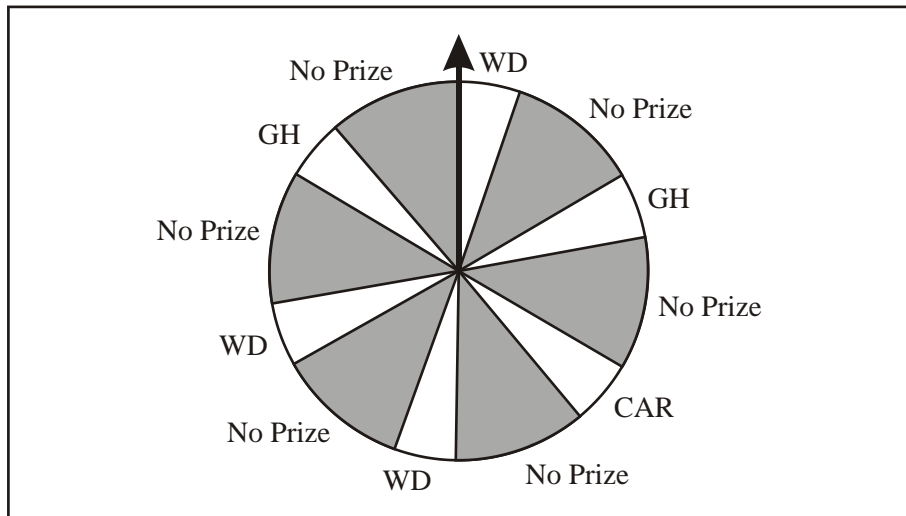
Answers:

(a)

(b)

(Total 4 marks)

21. On a certain game show, contestants spin a wheel to win a prize, as shown in the diagram. The larger angles are 40° (the shaded sectors), and the smaller angles are 20° .



Find the probability that a contestant

- (a) will **not** win a prize;
- (b) will win a holiday in Greece (GH);
- (c) will win a washer/dryer (WD), given that he knows that he has won a prize;

(d) will win a holiday in Greece **or** a washer/dryer.

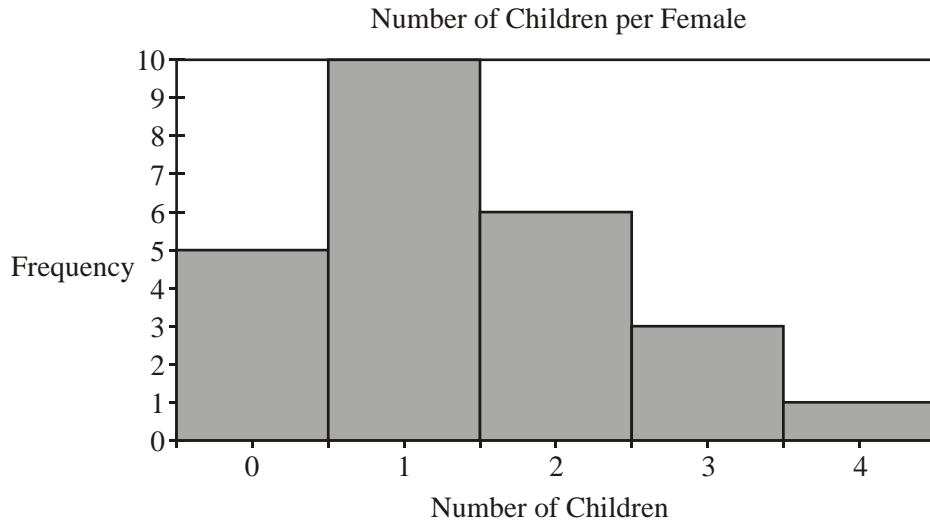
Working:

Answers:

- (a)
- (b)
- (c)
- (d)

(Total 4 marks)

22. A group of 25 females were asked how many children they each had. The results are shown in the histogram below.



- (a) Show that the mean number of children per female is 1.4. (2)
- (b) Show clearly that the standard deviation for this data is approximately 1.06. (3)
- (c) Another group of 25 females was surveyed and it was found that the mean number of children per female was 2.4 and the standard deviation was 2. Use the results from parts (a) and (b) to describe the differences between the number of children the two groups of females have. (2)

- (d) A female is selected at random from the first group. What is the probability that she has more than two children? (2)
- (e) Two females are selected at random from the first group. What is the probability that
- (i) both females have more than two children? (2)
- (ii) only one of the females has more than two children? (2)
- (iii) the second female selected has two children given that the first female selected had no children? (1)
- (Total 15 marks)**

23. A group of 30 children are surveyed to find out which of the three sports cricket (C), basketball (B) or volleyball (V) they play. The results are as follows:

3 children do not play any of these sports
 2 children play all three sports
 6 play volleyball and basketball
 3 play cricket and basketball
 6 play cricket and volleyball
 16 play basketball
 12 play volleyball.

- (a) Draw a Venn diagram to illustrate the relationship between the three sports played. (1)
- (b) On your Venn diagram indicate the number of children that belong to each region. (3)
- (c) How many children play only cricket? (2)
- (Total 6 marks)**

24. Consider the following statements:

p : Good mathematics students go to good universities

q : Good music students are good mathematics students

r : Students who go to good universities get good jobs

- (a) From these statements, write two **valid** conclusions.

(b) Write in words each of the following

(i) $\neg q$;

(ii) $p \wedge r$.

Working:

Answers:

(a)

.....

.....

(b) (i)

.....

(ii)

.....

(Total 4 marks)

25. Of a group of five students, two will be selected to visit the United Nations. The five students are John, Maria, Raul, Henri and Susan.

(a) With the aid of a tree diagram or a table of outcomes, find the number of **different** possible combinations of students that could go to the United Nations.

(b) Find the probability that both Maria and Susan will go on the trip.

Working:

Answers:

(a)

(b)

(Total 4 marks)

26. Three propositions are defined as follows:

p : The oven is working.

q : The food supply is adequate.

r : The visitors are hungry.

(a) Write one sentence, in words only, for each of the following logic statements.

(i) $q \wedge r \wedge \neg p$ (2)

(ii) $\neg r \vee (p \wedge q)$ (2)

(b) Write the sentence below using only the symbols p , q and logic connectives.

"If the oven is working and the food supply is adequate then the oven is working or the food supply is adequate."

(2)

(c) A tautology is a compound statement which is always true. Use a truth table to determine whether or not your answer to part (b) is a tautology.

Hint: Begin by writing the first two columns of your truth table in the following format:

p	q
T	T
T	F
F	T
F	F

(3)

(Total 9 marks)

27. Let

$S = \{\text{positive integers less than 15}\};$

$X = \{\text{multiples of 2}\};$

$Y = \{\text{multiples of 3}\}.$

(a) Show, in a Venn diagram, the relationship between the sets S , X and Y .

(1)

(b) List the elements of:

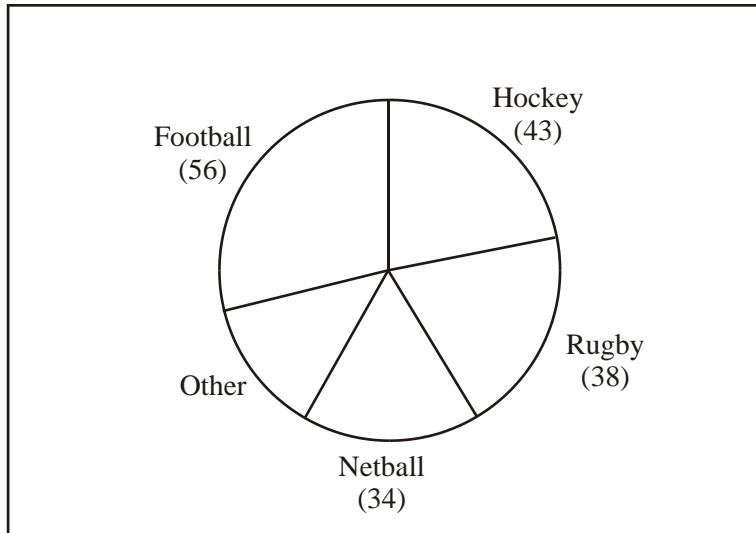
(i) $X \cap Y$ (1)

(ii) $X \cup Y$. (2)

(c) Find the **number of elements** in the complement of $(X \cup Y)$.

(2)
(Total 6 marks)

28. In a school, 180 pupils are asked which is their favourite outdoor sport in winter. The pie chart shows the result of the survey. The diagram is **not** accurately drawn.



(a) Calculate the angle of the sector representing rugby.

(b) Estimate the probability that a pupil's favourite outdoor sport in winter will be hockey.

Working:

Answers:

(a)

(b)

(Total 4 marks)

29. Three propositions p , q and r are defined as follows:

p : the water is cold. q : the water is boiling. r : the water is warm.

(a) Write one sentence, in words, for the following logic statement:

$$(\neg p \wedge \neg q) \Rightarrow r$$

(b) Write the following sentence as a logic statement using symbols only.

"The water is cold if and only if it is neither boiling nor warm"

Working:

Answers:

(a)

(b)

(Total 4 marks)

30. The table below shows the relative frequencies of the ages of the students at *Ingham High School*.

Age (in years)	Relative frequency
13	0.11
14	0.30
15	0.23
16	0.21
17	0.15
Total	1

- (a) If a student is randomly selected from this school, find the probability that
- (i) the student is 15 years old;
 - (ii) the student is 16 years of age or older.

There are 1200 students at *Ingham High School*.

- (b) Calculate the number of 15 year old students.

Working:

Answers:

- (a) (i)
- (ii)
- (b)

(Total 4 marks)

- 31.** A bag contains 2 red, 3 yellow and 5 green sweets.

Without looking, Mary takes one sweet out of the bag and eats it. She then takes out a second sweet.

- (a) If the first sweet is green, what is the probability that the second sweet is also green?
- (b) If the first sweet is not red, what is the probability that the second sweet is red?

Working:

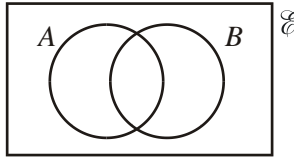
Answers:

- (a)
- (b)

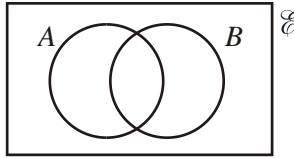
(Total 4 marks)

32. In each of the Venn diagrams, shade the region indicated.

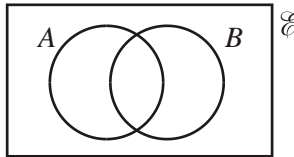
(a) $A \cap B$



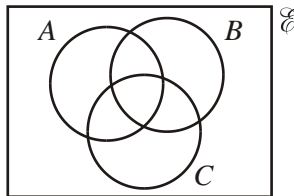
(b) The complement of $(A \cap B)$



(c) The complement of $(A \cup B)$



(d) $A \cup (B \cap C)$



Working:

(Total 4 marks)

33. *Note: For this question, it is important that you show your working and explain your method clearly.*

A box contains 10 coloured light bulbs, 5 green, 3 red and 2 yellow. One light bulb is selected at random and put into the light fitting of room A.

- (a) What is the probability that the light bulb selected is
- (i) green? (1)
 - (ii) not green? (1)

A second light bulb is selected at random and put into the light fitting in room B.

- (b) What is the probability that
- (i) the second light bulb is green given the first light bulb was green? (1)
 - (ii) both light bulbs were not green? (2)
 - (iii) one room had a green light bulb and the other room does not have a green light bulb? (3)

A third light bulb is selected at random and put in the light fitting of room C.

- (c) What is the probability that
- (i) all three rooms have green light bulbs? (2)
 - (ii) only one room has a green light bulb? (3)
 - (iii) at least one room has a green light bulb? (2)

(Total 15 marks)

34. The table below shows the number of left and right handed tennis players in a sample of 50 males and females.

	Left handed	Right handed	Total
Male	3	29	32
Female	2	16	18
Total	5	45	50

If a tennis player was selected at random from the group, find the probability that the player is

- (a) male and left handed;
- (b) right handed;
- (c) right handed, given that the player selected is female.

Working:

Answers:

- (a)
- (b)
- (c)

(Total 4 marks)

35. It is known that 5% of all AA batteries made by Power Manufacturers are defective. AA batteries are sold in packs of 4.

Find the probability that a pack of 4 has

- (a) exactly two defective batteries;
- (b) at least one defective battery.

(3)

(2)

(Total 5 marks)

36. Let $S = \{x : 1 \leq x < 17, x \in \mathbb{N}\}$.

P , Q and R are the subsets of S such that

$P = \{\text{multiples of four}\};$

$Q = \{\text{factors of 36}\};$

$R = \{\text{square numbers}\}.$

(a) List the elements of

(i)

(ii) $P \cap Q \cap R.$

(2)

(b) Describe in words the set $P \cup Q.$

(1)

(c) (i) Draw a Venn diagram to show the relationship between sets P , Q and $R.$

(2)

(ii) Write the elements of S in the appropriate places on the Venn diagram.

(3)

(d) Let p , q and r be the statements

$p : x$ is a multiple of four;

$q : x$ is a factor of 36;

$r : x$ is a square number.

(i) Write a sentence, in words, for the statement;

$(p \vee r) \wedge \neg q$

(2)

(ii) Shade the region on your Venn diagram in part (c)(i) that represents $(p \vee r) \wedge \neg q$

(1)

- (iii) (a) Use a truth table to determine the values of $(p \vee r) \wedge \neg q$. Write the first three columns of your truth table in the following format.

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

(3)

- (b) Write down one possible value of x for which $(p \vee r) \wedge \neg q$ is true.

(1)

(Total 15 marks)